Optimal Crumb Cleanup

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Problem Statement

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One- and Two-Sweep Lemmas

Lemma 1: One sweep (either orthogonal or slanted) will suffice if and only if all points are collinear.
Corollary 1: If 1 sweep suffices then the cost of the (optimal) sweep will equal the length of the line segment connecting the two points furthest from each other.
Lemma 2: If limited to 2 sweeps, the minimal cost will be half the perimeter of the minimal parallelogram.
Do two sweeps always suffice?

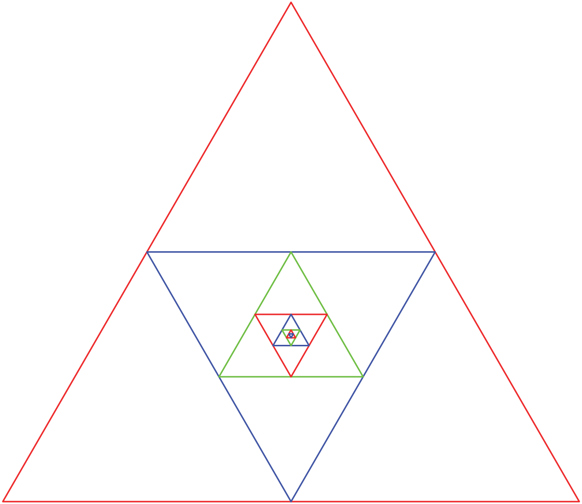
One-Flush Lemma

[Mitchell and Polishchuk, Minimum-Perimeter Enclosing k-gon, 2009.]

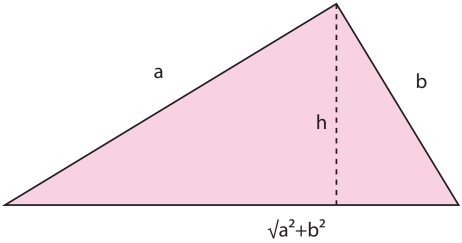
3-Sweep Example

[Square with diagonal antenna.]
Consider a 1x1 square with a diagonal of length $\sqrt{2}$ jutting out from one corner. By the 1-flush lemma, there are two ways of enclosing this shape in a parallelogram, namely (a) within a 2x2 square or (b) within a parallelogram formed by connecting the vertex of the diagonal with the two adjacent vertices of the square (this makes two sides of the figure, then reflect those sides to complete.) (a) has cost 4, and (b) cost $2\sqrt{5} = 4.47$ which are each greater than the cost of sweeping the diagonal and then the square $(2 + \sqrt{2} = 3.41)$, requiring at least 3 sweeps.

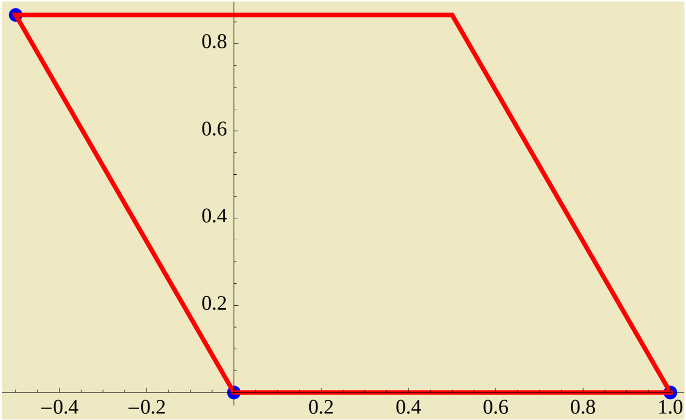
Two Sweeps Suffice for Triangles?



Right Triangles



Evidence for Two Sides Flush



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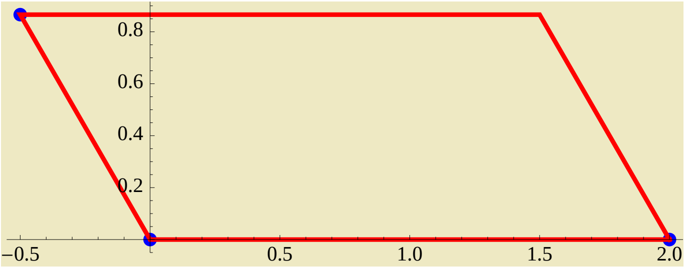

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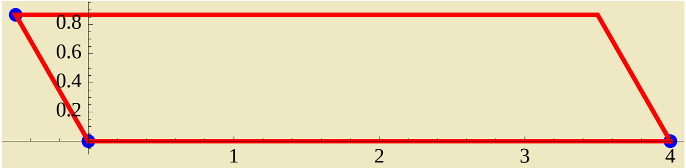
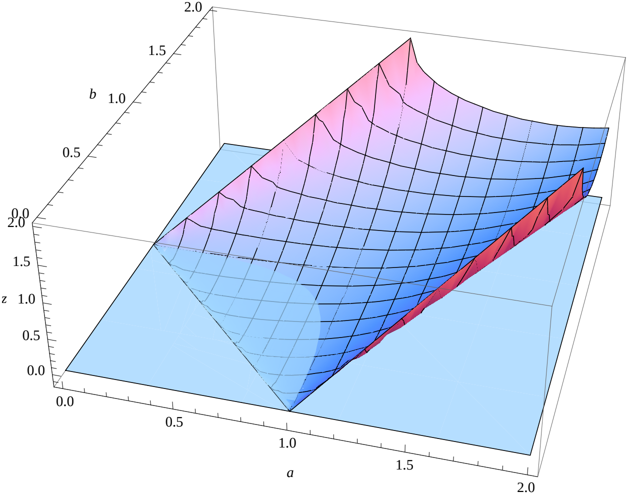
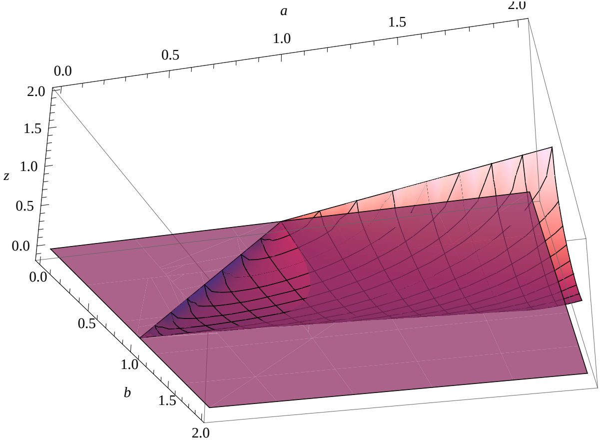


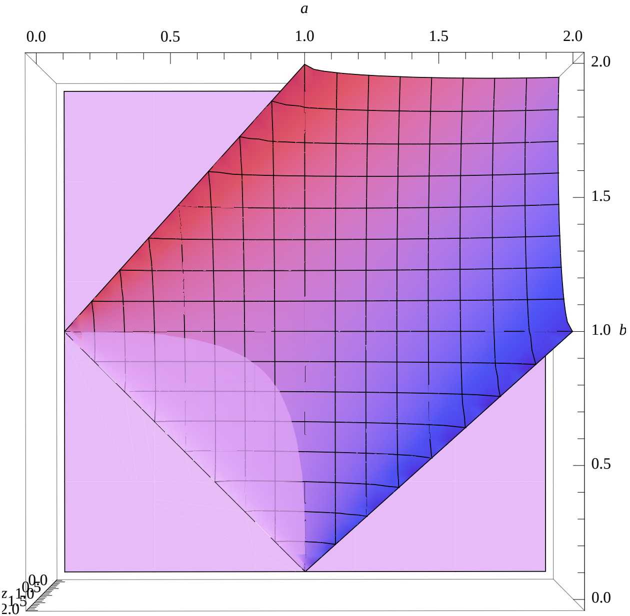
FlushFunction, Angled View



FlushFunction, Underneath



FlushFunction, Overhead View



What's Next?

- Lowerbound for two sweeps? Equilateral triangle: $\sqrt{3}$ vs. $1 + \sqrt{3}/2$.

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