Moving Coins (More Than Two)

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1 Introduction

Consider a collection of discs or *coins*. The coins are found resting on a plane surface so that no two overlap. We explore issues involved in moving the coins from their initial positions to some desired final position.

To be more precise, we can move a coin centered at point a to a position centered at point b if the trajectory of the coin along the line segment ab does not collide with another coin. We say that such a translation in one fixed direction is one *move*. We are given as input a set of coins $C = \{c_1, c_2 \dots c_n\}$ positioned at initial source locations $P = \{p_1, p_2, \dots p_n\}$ and a set of final destinations $Q = \{q_1, q_2, \dots q_n\}$, where P and Q are sets of points. Associated with each coin c_i is an *agenda* $a_i \subseteq Q$ a set of possible destinations. As output we need to produce an *itinerary*, an ordered list of moves satisfying the agenda for every coin. The objective is to produce an efficient itinerary. The *cost* of an itinerary is simply the number of moves used.

This problem is motivated by measuring the difference between various configurations. For example one can measure the difference between two strings of text by their *edit distance* [1]. The edit distance is the minimum number of text editor operations needed to go from one string to another. A distance with a more geometric flavour is the *earth movers distance* [2]. The earth movers distance measures the minimum amount of work needed to go from one configuration to another. The notion of work is flexible and conforms to the application. Thus our problem of moving coins is in the same vein as the previous examples. We are interested in the minimum number "move" operations needed to go from one configuration of coins to another.

Our problem can also be viewed as a simplified model of multi-robot path planning. Consider a collection of robots, whose footprints are discs, manoeuvring in a common workspace. A robot's tasks may take it from one destination to another. Our notion of an agenda for a coin is a simplified way to model this type of situation. A survey paper by Hwang and Ahuja [3] discusses the general robot path planning problem and multi-robot path planning in particular.

Erik and Martin Demaine with Helena Verrill [4] examine sliding coin puzzles, that is, moving coins from one configuration to another subject to two coins always being in contact. They present theorems on the solubility of such puzzles, and algorithms to produce optimal solutions when they exist. They also include references to other similar puzzles.

We consider several different variations on our coin moving problem. We first consider the case where $a_i = Q$ for all coins in C, that is each coin may go to any of the destinations, furthermore the coins are all of the same radius. We show that an itinerary of cost $2n - 1$ is always sufficient, and an itinerary of cost $3n/2$ is sometimes necessary. When the coins are of various diameters, and the destination position of each coin is guaranteed not to overlap with other coins in their destination positions, we obtain tight bounds. We show that an itinerary of cost $2n$ is sometimes necessary and always sufficient. We then consider the coin placement problem, that is, determining whether the set of destinations can accommodate all of the coins without overlap. We show that deciding a non-overlapping coin placement is NP-complete.

Our algorithms for obtaining the sufficiency bounds use moves of unbounded distance. Thus we also consider cases where the moves are confined to a small workspace. Suppose that each coin is of unit diameter, and the sources and destinations lie within a rectangular bounding box. We want to satisfy the agendas with an itinerary that keeps all coins within the bounding box. We show that an itinerary of cost $3n$ is always sufficient if one side of the rectangle is at least of length n. When the coins are of various diameters are confined to a similarly confining workspace we show that an itinerary of cost $4n$ always suffices. We then consider other confined workspace settings and show how various workspace areas and aspect ratios affect the cost of the itinerary.

All of our lower bound algorithms make use of some type of sorting strategy to organize the moves. This sorting step seems to be necessary because we show a variant of the coin problem is linear time reducible to sorting.

Subsequently we consider the case where a_i consists of exactly one destination for all coins in C. If the sources and destinations don't overlap then we can decide in $O(n^2)$ time whether there is an itinerary of cost n. At the other end of the spectrum if we allow the agendas to consist of two or more destinations we show that deciding whether there is an itinerary of cost n is NP-complete.

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References

1. Levenshtein, V.I.: Binary codes capable of correcting deletions, insertions and reversals. Soviet Phys. Dokl **10** (1966) 707–710
2. Rubner, Y., Tomasi, C., Guibas, L.J.: The earth movers distance as a metric for image retrieval. International Journal of Computer Vision **40** (2000) 99–121
3. Hwang, Y., Ahuja, N.: Gross motion planning- a survey. ACM Computing Surveys **24** (1992) 219–291
4. Demaine, E., Demaine, M., Verrill, H.: Sliding coin puzzles. In Nowakowski, R.J., ed.: More Games of No Chance. Cambridge University Press, Collection of papers from the MSRI Combinatorial Game Theory Research Workshop, Berkeley, California (2002) 405–431