YOU MAY USE TWO 8.5"x11" SHEETS OF NOTES ON THIS EXAM.
YOU MAY NOT USE THE TEXTBOOK, A COMPUTER, OR ANY OTHER INFORMATION
SOURCE BESIDES YOUR TWO PAGES OF NOTES.

All work should be written in the exam booklet. Partial credit will be granted where appropriate if intermediate steps are shown. When you are done, please sign the statement below. Good luck!
1. Graph Traversal (12 points)

a.) Consider the directed graph at right. Simulate Dijkstra’s shortest path algorithm starting at node H. In your results, give (i) the order in which the nodes are visited, (ii) all the cost labels computed for each node as the algorithm runs, and (iii) the homeward pointing edge associated with each of those costs. (For example, for node X you might say that the initial infinite cost was replaced with a cost of 12 via node Y, then with a cost of 8 via node Z, and finally with a cost of 7 via node W.)

b.) For which starting nodes could node E appear in the fourth position of a breadth-first traversal? You may break ties in any manner you please. For example, BFAECGDFH is a valid breadth-first traversal that starts at B and has E in the fourth position, so you would list B as part of your answer.

2. Trees (12 points)

a.) Consider the tree at right. Draw the tree that would result from a left-rotation of the root.

b.) List the order in which the tree nodes would be visited in a postorder traversal (for the original tree).

c.) What changes would you need to make to the original tree, if any, to make it a valid binary search tree (assuming the ordering relation on nodes is alphabetical)?

3. Programming (8 points)

Rewrite the snippets of Java code below either to make them more efficient or more in line with the style guidelines promoted in this course. The effect should remain unchanged.

a.)
```java
if (x==7) {
    return true;
} else {
    return false;
}
```

b.)
```java
public int addOne(int x) {
    return x+1;
}

public int addTwo(int x) {
    return x+2;
}

public int addThree(int x) {
    return x+3;
}
```
4. **Hash Tables** *(12 points)*

Consider the hash table shown below, which uses the simple hash function \( h(k) = k \mod 7 \) and handles collisions via simple linear probing. Answer the questions that follow.

<table>
<thead>
<tr>
<th>Key</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>49</td>
<td>Tango</td>
</tr>
<tr>
<td>13</td>
<td>Alpha</td>
</tr>
<tr>
<td>73</td>
<td>Charlie</td>
</tr>
<tr>
<td>24</td>
<td>Bravo</td>
</tr>
<tr>
<td>20</td>
<td>Foxtrot</td>
</tr>
</tbody>
</table>

a.) List all the (key,value) pairs that are not stored at their home position in the table.

b.) List all the (key,value) pairs which, if removed, would cause other (key,value) pairs to change their position.

c.) If the table was initially empty, and the (key,value) pairs you see were added in some sequence without any other intervening operations, what can you infer about their relative order of insertion? List all sets of (key,value) for which you can determine that one must have been inserted before the other, and give the ordering.

```java
c.
public class Cheer {
    public static int cheerNumber;

    public static void method1() {
        cheerNumber = (int) Math.floor(Math.random() * 3) + 1;
    }

    public static void method2() {
        for (int i = 0; i < cheerNumber; i++) {
            System.out.println("Hip Hip Hooray!");
        }
    }

    public static void main(String[] args) throws IOException {
        method1();
        method2();
    }
}
d.
BufferedReader input = new BufferedReader(new InputStreamReader(System.in));
System.out.println("Please enter your age in years.");
String temp1 = input.readLine();
int temp2 = Integer.valueOf(temp1) * 12;
System.out.println("That is "+temp2+" months!");
```
5. **Program Analysis** (16 points)

Give the asymptotic runtime performance ("big-O notation") of the following sets of operations, in terms of the problem size $n$.

a.) All the elements in an unsorted list of length $n$ are added one at a time to a rebalancing binary search tree (such as a red-black tree).

b.) All the elements in a sorted array of length $n$ are added to an ordinary binary search tree using a recursive method that adds the middle element of the array at the root, and then recursively builds the left and right subtrees using the left and right halves of the array.

c.) An unsorted array is converted to a heap in place. Next the heap is shrunk by popping the largest element and moving it to the vacated point in the array, as shown in the picture at right.

d.) Given an array of $n$ different integers $a_0...a_{n-1}$, a double nested loop examines each pair $(a_i,a_j)$ to determine whether they are relatively prime. (Assume for simplicity that testing relative primeness is a constant-time operation.) The result is a boolean value, which is stored in a hash table using the pair of numbers as the key.

6. **Recursion** (12 points)

Consider the recursive method in the box.

```java
public static void printRecursive(int x) {
    if (x > 0) printRecursive(x-1);
    System.out.print(x+" ");
    if (x > 0) printRecursive(x-1);
}
```

a.) What output would be generated by a call to `printRecursive(4)`?

b.) How many times in total does `printRecursive` get called in the course of executing `printRecursive(4)`?

c.) If the second recursive call (the last line of the method body) is commented out, what output would be generated by a call to `printRecursive(4)`?

7. **Abstract Data Types** (8 points)

Give the abstract data type description for a list iterator: Describe the operations it must implement, any data it must keep track of, the effects of the operations on the data, and the inputs and outputs for the operations. You may omit the operations that modify the list.
8. Inheritance (8 points)

Consider the program at right. For each item below, write a line of Java code that you could place within the main() method to accomplish the task described, or write “not possible” if no such command exists. Use casts only when necessary.

a.) Cause n1.toString() to return “Crow Hen” when called from main().

b.) Cause n1.toString() to return “Robin Hen” when called from main().

c.) Cause n1.toString() to return “Dove Velociraptor” when called from main().

d.) Cause n2.toString() to return “Dove Squirrel Auk Kiwi” when called from main().

e.) Cause n2.toString() to return “Dove Hen Squirrel Kiwi” when called from main().

f.) Cause n2.toString() to return “Dove Velociraptor Auk Kiwi” when called from main().

g.) Cause n2.toString() to return “Dove Hen Velociraptor Kiwi” when called from main().

h.) Cause n2.toString() to return “Dove Hen Auk Squirrel” when called from main().

9. Sorting (12 points)

Write a short essay summarizing all the sorting algorithms we have studied this semester: their asymptotic runtime, a brief description of how they work, and any distinctive features (what kind of extra storage space they require, if any, and whether they can perform a stable sort, for example).