Separation Sensitive Kinetic Collision Detection for Simple Polygons

David Kirkpatrick Bettina Speckmann
Department of Computer Science
University of British Columbia

Abstract

We extend the kinetic data structure for collision detection between moving simple polygons introduced in [2] to incorporate a two-layered hierarchical representation of convex chains. This permits us to define an adaptive hierarchical outer approximation for simple polygons. The latter can be exploited to give separation sensitive complexity bounds for kinetic collision detection comparable to those of Erickson et al. [1] but in a substantially richer setting (collections of simple polygons as opposed to pairs of convex polygons).

1 Introduction

Several kinetic data structures have been introduced for maintaining a certificate of separation (equivalently detecting collisions) for collections of polygonal objects in motion.

The kinetic separation structure (KSS) introduced in [2] maintains an active set of certificates whose number is proportional to a minimum size set of separating polygons for the objects. However, this structure as described is unable to exploit situations in which neighboring objects are widely separated (see Fig. 1 for an example where the convex chains within a pseudo triangle contain a large number of vertices in close proximity relative to their separation; this could result in a large number of events whenever the adjacent partition triangle moves). In such situations it is natural to approximate objects by some kind of coarse outer approximations whose disjointness certifies the separation of the underlying objects but is easier to maintain. For convex objects (viewed as the intersection of half spaces determined by their bounding edges) in isolation it is possible to construct a hierarchy of successively coarser approximations based on the simple idea of progressive relaxation of half space constraints. For convex objects in the context of other convex objects or, more generally, for collections of disjoint but possibly interleaved non-convex objects it is not immediately clear what constitutes a useful notion of hierarchical representation or how such a notion could be exploited to provide separation sensitive complexity bounds for kinetic collision detection. We show that with the imposition of two

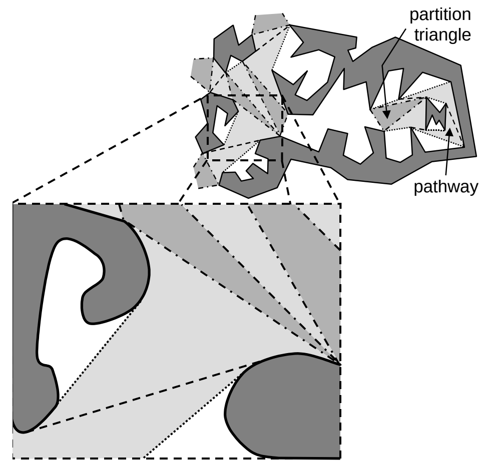

Figure 1: The convex chains of the pseudo triangles can consist of a large number of vertices.

new invariants on the pseudo triangulation of free space maintained in the KSS (which also happen to facilitate a simpler implementation) we can replace convex chains within pseudo triangles by hierarchical approximation structures that lead to fewer certificate update events as well as more efficient implementations of the pseudo triangulation update primitives that form the core of the KSS.

The resulting approximation structure is adaptive in the sense that it yields a representation of the moving polygons that on the pathways separating neighbouring objects contains only a number of vertices proportional to a minimum size set of separating polygons for the objects independent of their actual separation.

As the objects move the structure shifts its level of detail so as to provide a closer approximation in areas where the polygons are close to each other while coarsening in areas where the polygons are well separated. We construct and maintain the approximation conforming to the partition of the pathways into pseudo triangles. This automatically provides us with the means to detect the parts of the current boundary of the polygons where we can extend the approximation or where we have to increase its level of detail. The total number of vertices on the pathways remains constant during those operations.

A complete analysis of the efficiency of our modified structure (like that of the unmodified KSS structure) remains a challenge. It is possible to provide some partial results that demonstrate the competitiveness of this structure with the separation sensitive structure of Erickson et al. that was designed for maintaining the separation of *two convex* objects. Specifically, for two neighbouring but separable polygons in the plane let D be their maximum diameter and n be the total number of vertices. Let s be the minimum distance between their convex hulls during their motion. Our structure processes $O(\log(D/s))$ events when the relative motion of the two polygons is along a straight line, $O(D/s)$ events when the relative motion of the two polygons is along a convex curve. All events can be processed at a cost of $O(\log(D/s))$ per event.

2 Definitions and Invariants

We propose a simply hierarchy that creates *approximation vertices* by iteratively extending adjacent edges of a convex *base chain*, allowing the four configurations depicted in Fig. 2. See Fig. 3 for an example.

This construction maintains the following invariant:

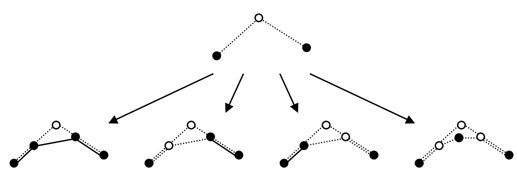

Figure 2: The four different possible configurations.

Invariant 1 *The base chain underneath an approximation vertex is convex and ends at two polygon vertices. Furthermore no two approximation vertices are adjacent to each other.*

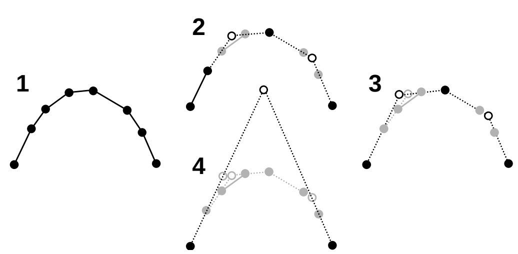

Figure 3: Constructing the hierarchy.

The hierarchy consists of two layers. The top layer is balanced by angle, the bottom one by number of vertices, yielding a total maximum depth of at most $O(\log n)$.

The hierarchy supports two operations:

1. Join two convex chains that together form a convex chain and merge the hierarchy on top of them. This operation only requires changes to the top layer of the hierarchy and the cost is therefore proportional to the difference in height between the two hierarchies.
2. Add or extract one polygon edge from the hierarchy. This operation is only required in conjunction with corner triangles and costs $O(\log n)$.

Two additional invariants on the shape of object and corner triangles in a pathway guarantee that those two operations are sufficient to maintain our structure.

3 Maintenance

Partition triangles and bridges do not distinguish between polygon and approximation vertices. All update operations as described in [2] can therefore be executed as before, the only additional required work is a possible restructuring step in the hierarchies of the pseudo triangles whenever a complete base chain moves from one triangle into the next. This can be done in $O(\log n)$ time.

The hierarchy however introduces two new events: (i) a polygon edge or chord collides with an approximation vertex, and (ii) a polygon vertex collides with an edge that ends on an approximation vertex. Both types of events can be handled in $O(\log n)$ time by "popping" the approximation vertex and restoring the invariants using the convex chain that was previously hidden. See Fig. 4 for an example and note how the hierarchy shifts after the collision.

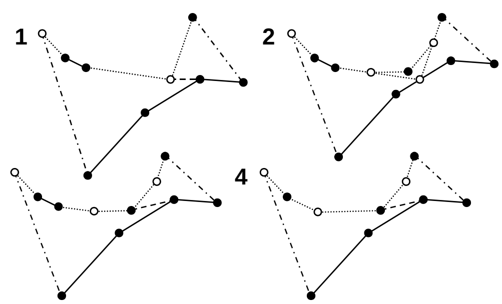

Figure 4: A polygon edge or chord collides with an approximation vertex.

References

[1] J. Erickson, L. Guibas, J. Stolfi, and L. Zhang. Separation-sensitive collision detection for convex objects. In *Proc. 10th ACM-SIAM Sympos. Discrete Algorithms*, pages 327–336, 1999.

[2] D. Kirkpatrick, J. Snoeyink, and B. Speckmann. Kinetic collision detection for simple polygons. In *Proc. 16th ACM Sympos. on Comp. Geom.*, pages 322–330, 2000.